THE EFFECTS OF SEDIMENT TRANSPORT ON GRAIN-SIZE DISTRIBUTIONS

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ABSTRACT: Changes in statistics (mean, sorting, and skewness) describing grain-size distributions have long been used to speculate on the direction of sediment transport. We present a simple model whereby the distributions of sediment in transport are related to their source by a sediment transfer function which defines the relative probability that a grain within each particular class interval will be eroded and transported. A variety of empirically derived transfer functions exhibit negatively skewed distributions (on a phi scale). Thus, when a sediment is being eroded, the probability of any grain going into transport increases with diminishing grain size throughout more than half of its size range. This causes the sediment in transport to be finer and more negatively skewed than its source, whereas the remaining sediment (a lag) must become relatively coarser and more positively skewed.

Flume experiments show that the distributions of transfer functions change from having a highly negative skewness to being nearly symmetrical (although still negatively skewed) as the energy of the transporting process increases. We call the two extremes low-energy and high-energy transfer functions, respectively. In an expanded sediment-transport model, successive deposits in the direction of transport are related by a combination of two transfer functions. If energy is decreasing and the transfer functions have low-energy distributions, successive deposits will become finer and more negatively skewed. If, however, energy is increasing, but the initial transfer function has a high-energy distribution, successive deposits will become coarser and more positively skewed.

The variance of the distributions of lags, sediment in transport, and successive deposits in the down-current direction must eventually decrease (i.e., the sediments will become better sorted). We demonstrate that it is possible for variance first to increase, but suggest that, in reality, an increasing variance in the direction of transport will seldom be observed, particularly when grain-size distributions are described in phi units.

This model describing changes in sediment distributions was tested in a variety of environments where the transport direction was known. The results indicate that the model has real-world validity and can provide a method to predict the directions of sediment transport.

INTRODUCTION

The environmental interpretation of grain-size distributions found in sedimentary deposits has been, and still is, a fundamental goal of sedimentology. Ever since Udden’s work in 1914 it has been recognized that sediment size fractions approximate a log-normal distribution. In reality, however, most sediments do not strictly follow log-normality and deviations from the Gaussian model have been given various environmental interpretations (Folk and Ward 1957; Mason and Folk 1958; Friedman 1961, 1979). Other workers have differentiated log-normal subpopulations defined by distinct breaks within the complete size range of the sample (Moss 1962; Visher 1974; Glaister and Nelson 1974; Middleton 1976). Each subpopulation is interpreted as representative of a specific transport mode (traction, intermittent suspension, and suspension), the relative concentrations of each suggesting particular depositional environments.

Although Blatt et al. (1980) argue that it should not be surprising for specific process populations to exist because different transport mechanisms differ in the way they select grains for movement or deposition, many other authors are unable to achieve similar interpretations (Garraw 1982; Flemming 1982; Anderson et al. 1982). There are at least two reasons for such disagreements. First, the processes of erosion and deposition encompass complex functions involving many variables, including grain orientation and the structure of turbulence, which are intrinsically random (Gessler 1976). For example, there is substantial disagreement as to the actual velocities required to erode and transport particles of a given size, particularly when the bed material is poorly sorted (Singer and Anderson 1984). Experimental difficulties appear to be as great today as in 1950 when Einstein wrote, “The forces acting on individual particles of a natural sediment mixture in a bed cannot very well be measured” (Einstein 1950, p. 35). Both Singerland (1977) and Singer and Anderson (1984) provide good summaries of the complexities encountered by workers who have sought to document the processes of entrainment and deposition of sediments.

The second difficulty lies in the imprint of the source sediment characteristics on the characteristics of the deposit. This was recognized as long ago as 1938 when Krumbein suggested the importance of progressive or continuous changes in grain-size distributions from source to final deposit (Krumbein 1938). Progressive changes have been recognized by several workers (Stapor and Tanner 1975; McCave 1978; Haner 1984) and have been analyzed in a deductive model by McLaren (1981). He suggested that the mean, sorting, and skewness of grain-size frequency distributions follow trends that identify the direction of transport and the sedimentary processes of winnowing, selective deposition, and total deposition. Using a hypothetical sediment distribution and an as-

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1 Manuscript received 8 February 1984; revised 16 November 1984.
summation that light grains have a greater probability of being eroded and transported than heavy grains, the model demonstrated that 1) sediment in transport must be finer, better sorted, and more negatively skewed than its source sediment; 2) a lag must become coarser, better sorted, and more positively skewed; and 3) successive deposits may become finer or coarser, but the sorting must become better and the skewness more positive.

In this paper we attempt to scrutinize the validity of these trends in a more rigorous examination than the strictly deductive approach used by McLaren (1981). We present a more refined model that demonstrates how grain-size distributions of sedimentary deposits change in the direction of transport. The paper does not, however, provide solutions or increase our understanding of transport and depositional processes; rather, it recognizes that the nature of these processes is probabilistic, and their results should be reflected in the relative changes of grain-size distributions found in interrelated sedimentary deposits. The model is tested in a variety of environments, and we propose a technique to interpret the observed changes in grain-size distributions for the purpose of establishing sediment-transport directions.

Initial Sediment Transport Model

Let us consider any grain-size distribution (g(s)) (Fig. 1). If eroded, the sediment which is transported has a new distribution (r(s)) which is derived from g(s) according to a function t(s) so that
\[ r(s_i) = kg(s_i)t(s_i) \]
or
\[ t(s_i) = \frac{r(s_i)}{kg(s_i)}, \tag{1} \]
where g(s) and r(s) define the proportion of the sediment in the ith grain-size class interval for each of the sediment distributions. k is a scaling factor that normalizes r(s) so that
\[ \sum_{i=1}^{N} r(s_i) = 1. \]
Thus,
\[ k = \frac{1}{\sum_{i=1}^{N} g(s_i)t(s_i)}. \]

With the removal of r(s) from g(s), the remaining sediment (a lag) has a new distribution denoted by d(s) (Fig. 1), where
\[ d(s_i) = k'g(s_i)(1 - t(s_i)) \]
or
\[ t'(s_i) = \frac{d(s_i)}{k'g(s_i)}, \]
where
\[ t'(s_i) = 1 - t(s_i). \]

As well as using distribution functions to characterize sediment size, we use t(s) in a similar manner to describe statistically all of the processes which tend to move sediment from one location to another. This function is described in the same manner as a size distribution function (i.e., by weight proportion, grain frequency, etc.), and each t(s_i) gives the probability of transferring grains of size "s_i" from a source (g(s)) into transport (r(s)). We propose to call each t(s_i) a transfer coefficient and t(s) a sediment-transfer function. The latter may be thought of as a function that incorporates all sedimentary and dynamic processes that result in initial movement and transport of particular grain sizes during a period of time.

Implicit in the earlier model proposed by McLaren (1981) was the assumption that light grains have a greater probability of being transferred from a source into transport than heavy grains. This suggests that t(s) is a function which increases (on a phi scale) or decreases (on a millimeter or linear scale) monotonically; that is, smoothly and continuously.\(^2\) Although this concept of t(s) is a simplification, the model derived from this assumption has proven to be effective in practice for determining sediment-transport paths and the relationships among interrelated sedimentary deposits (see, for example, McLaren et al. 1981; McLaren 1981, 1982, 1983, 1984).

If we assume that t(s) is monotonically increasing, it can be shown that the sediment in transport (r(s)) is always finer and more negatively skewed than its source (g(s)) and that the lag (d(s)) is always coarser and more positively skewed than its source (g(s)). The trend in sorting depends on the initial sediment distribution of the source and therefore cannot be determined for any g(s). Sorting must, however, eventually become better in both the sediment in transport and in the lag. The proof for these results is not essential to the text of this paper and is shown in the Appendix.

Expanded Sediment-Transport Model

We have shown in the Appendix how grain-size distributions of sediment in transport and the remaining lag deposit must change relative to a common source sediment, assuming that the sediment-transfer function is monotonically increasing. We now wish to (1) expand the simplified model shown in Figure 1, and (2) examine the validity of the assumption (i.e., the nature of t(s) in light of empirical data).

\(^2\) The discussion on the use of phi versus metric continues (see McManus 1982). We find that the log-normal transformation to phi provides better statistical descriptors (i.e., mean, sorting, and skewness) for the identification of sediment-transport directions than those based on a linear scale. The remainder of the paper uses the phi scale exclusively.
Consider a unique sediment source such as an eroding cliff of unconsolidated sediments with a grain-size distribution \( g(s) \) (Fig. 2). Eroded sediments are deposited in a down-current direction, forming a beach whose grain-size distributions are \( d_1(s) \), \( d_2(s) \), \( d_3(s) \), \ldots, respectively. The sediments in transport are denoted by \( r_1(s) \), \( r_2(s) \), \ldots. Let the first transport function \( t_1(s) \) be equal to 1 for all \( s \) so that \( r_1(s) = g(s) \). This is analogous to a sudden mass wasting event on the cliff face whereby the complete distribution \( g(s) \) is momentarily in transport. This distribution \( r_1(s) \) is then acted upon by a process represented by the function \( t_2(s) \) which results in a new distribution in transport, \( r_2(s) \). The sediment remaining is deposited as \( d_2(s) \) which is related to \( r_2(s) \) by the function \( 1 - t_2(s) \). Similarly, \( r_3(s) \) is acted upon by \( t_3(s) \) with the result that \( d_3(s) \) is deposited.

Any three boxes forming an equivalent pattern to those considered in Figure 1 (e.g., \( r_1(s) \), \( r_2(s) \), and \( d_3(s) \); Fig. 2) can be analyzed in the manner outlined in the previous section and the Appendix using the assumption that \( t(s) \) is monotonically increasing. Therefore, sediments in transport \( r_1(s) \), \( r_2(s) \), \( r_3(s) \), \ldots, etc.) must become progressively finer and more negatively skewed, and each \( d(s) \) can be considered a lag of its corresponding sediment in transport \( r(s) \). Thus, for example, \( d_3(s) \) is coarser and more positively skewed than \( r_3(s) \).

We now wish to determine the relative changes in sediment distributions among the sequential deposits \( d_1(s) \), \( d_2(s) \), \( d_3(s) \), \ldots, bearing in mind that \( r(s) \), \( t(s) \), and \( f(s) \) are not observable.

Let us suppose that \( d_2(s) \) is related to \( d_1(s) \) by a function \( X(s) \), so that

\[
d_2(s) = kd_1(s)X(s)
\]

where

\[
k = \frac{1}{\sum_{i=1}^{N} d_i(s)X(s)}
\]

or

\[
X(s) = \frac{d_2(s)}{kd_1(s)}
\]

As illustrated in Figure 2, \( d_2(s) \) can also be related to \( d_1(s) \) by

\[
d_2(s) = \frac{kd_1(s)t_1(s)(1 - t_1(s))}{1 - t_1(s)} = kd_1(s)X(s),
\]

where

\[
X(s) = \frac{t_1(s)(1 - t_2(s))}{1 - t_1(s)}.
\]

\[\text{[2]}\]

\( X(s) \) is a function which combines the effects of the two transfer functions \( t_1(s) \) and \( t_2(s) \). As such, \( X(s) \) may also be considered a transfer function in that it provides the statistical relationship between two sequential deposits. Similar to \( t(s) \), this function incorporates all of the processes responsible for sediment transport and deposition resulting in a sequence of sedimentary deposits over the period of time represented by the samples. Therefore, the relative change in the distributions between \( d_2(s) \) and \( d_1(s) \) is dependent on the shape of the function \( X(s) \) which can be determined by examining empirically derived \( t(s) \) functions.

The Shape of \( t(s) \)

Transfer functions were calculated from data in Day (1980), Emmett et al. (1980), Ghosh et al. (1979) and Gibbs and Neill (1972). These data sets all produced similarly shaped curves in spite of a wide range of grain-size distributions; however, for illustrative purposes we will use data from flume experiments described by Day (1980), which are the most complete for our purposes.

These experiments were conducted in a 2.46-m-wide
Table 1.—Distribution of sediment in transport with increasing flow rates (m s⁻¹) and the respective transfer functions (data from Day 1980, Series A)

<table>
<thead>
<tr>
<th>Grain Size</th>
<th>Bed Material wt %</th>
<th>Sediment in Transport (wt %)</th>
<th>Transfer Functions (sediment-transfer coefficients)</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ (mm)</td>
<td>t(s) (0.56 m s⁻¹)</td>
<td>t(s) (0.61 m s⁻¹)</td>
<td>t(s) (0.66 m s⁻¹)</td>
</tr>
<tr>
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<td>0.12</td>
</tr>
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<td>1.05</td>
<td>0.71</td>
</tr>
<tr>
<td>2.50</td>
<td>4.46</td>
<td>8.00</td>
<td>6.01</td>
</tr>
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</tr>
<tr>
<td>1.50</td>
<td>5.68</td>
<td>13.57</td>
<td>9.94</td>
</tr>
<tr>
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<td>5.75</td>
<td>10.35</td>
<td>8.58</td>
</tr>
<tr>
<td>1.00</td>
<td>4.31</td>
<td>7.33</td>
<td>6.33</td>
</tr>
<tr>
<td>0.75</td>
<td>2.69</td>
<td>5.21</td>
<td>4.58</td>
</tr>
<tr>
<td>0.50</td>
<td>2.56</td>
<td>4.09</td>
<td>4.03</td>
</tr>
<tr>
<td>0.25</td>
<td>2.21</td>
<td>3.21</td>
<td>3.46</td>
</tr>
<tr>
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<td>4.98</td>
<td>4.73</td>
<td>5.87</td>
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<td>2.35</td>
<td>3.16</td>
</tr>
<tr>
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<td>6.96</td>
<td>4.64</td>
<td>6.73</td>
</tr>
<tr>
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<td>10.10</td>
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<td>8.87</td>
</tr>
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<td>7.49</td>
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</tr>
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<td>-2.67</td>
<td>6.87</td>
<td>1.05</td>
<td>2.83</td>
</tr>
<tr>
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<td>0.17</td>
<td>0.70</td>
</tr>
<tr>
<td>-3.25</td>
<td>2.27</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>&lt;3.25</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean: -0.31, 1.04, 0.51, 0.16, -0.37
Sorting: 1.84, 1.18, 1.46, 1.58, 1.59
Skewness: 0.20, -0.85, -0.45, -0.19, 0.24

A: Average of runs 3 and 5.
B: Average of runs 6 and 8.
C: Average of runs 7 and 9.
D: Average of runs 10 and 11.

NOTE: Each sediment in transport distribution was derived from two runs with closely similar mean velocities.

1 Anomalous skewness value possibly the result of inadequate sampling of fines at high flow rates (Day, pers. comm., 1984).

Recirculating flume with a sediment-return system for both suspended and bed-sediment loads. We will examine the data from the Series A experiments which utilized a bed material that ranged from ~3.25 φ to 4.0 φ, and was both poorly sorted (1.81 φ) and bimodal (Table 1). Series A consisted of 11 separate runs in which discharge, depth, water-surface slope, and mean velocity of the flow were controlled. The sediment in transport was sampled by means of collection baskets as it returned to the upstream end of the flume. We will look at the average weight percent distributions of sediment in transport and the resultant transfer functions, as calculated from Equation 1 at four mean flow velocities (Table 1).

The values of each k(t(s)) shown in Table 1 provide a measure of the relative probability of transport for each particular grain size. However, the absolute probability (t(s)) cannot be calculated directly with the given data, as the absolute weights of each sediment size rather than the weight percentages are required to do this. By definition, t(s) must be less than 1 (because it is a probability), and we assume that the probability of transporting any specific grain size increases with increasing flow rate. Considering these two factors, the absolute values of t(s) have been estimated by assuming a different value for k for each flow rate (Table 2). Although this process is somewhat arbitrary, the shape of the distributions remains the same, and the t(s) values now reflect an increasing proportion of each grain size going into transport as the flow rate increases (i.e., t(s)ₐ < t(s)ₙ < t(s)ₙ₋₁ < ...). Also, we wish to point out that, although each t(s) curve (Fig. 3) is a probability function, it is not a probability density function in that it does not define the probability of the occurrence of all possible events. Therefore, the area under the curve does not necessarily equal 1.

When graphed (Fig. 3), it is clear that the calculated t(s) functions are not monotonically increasing, as was assumed in the proof of the initial transport model (Appendix). Rather, each t(s) is an asymmetrical curve that rises to a peak before falling back to zero. Our assumption that fine grains are more easily transferred than coarse grains encompasses a second assumption which is hidden; namely, that the transfer of any particular grain size is independent of other grain sizes. A variety of processes, such as shielding in which fines are protected from movement by larger clasts, or the decreasing ability of the eroding process to carry additional fines with increasing load clearly invalidates this hidden assumption.

This observation, that sediment transport is dependent not only on grain size, but also on the interaction among the different grain sizes present, appears to have two significant effects. First, as stated previously, the transfer functions do not increase monotonically throughout the complete size distribution of the sediment source; and second, the position of the transfer function on the ab-
Fig. 3.—Weight percentage of sediment distribution of bed material (histogram) and the resultant transfer functions (from Table 2) under different flow regimes (from data in Day 1980).

The abscissa (Fig. 3) is dependent on the size distribution of the source sediment. For example, we have derived transfer functions associated with sediments of different distributions and ranges of grain sizes from the example shown here; however, the transfer functions were of the same general shape, the difference being their relative position on the abscissa, which shifted according to the range of grain sizes in the bed material.

In spite of the failure of the transfer functions to increase monotonically over the complete distribution of g(s), each t(s) is a negatively skewed curve, indicating that the function does increase over more than half of the grain-size distribution present in the bed material (Fig. 3). The fact that in nearly all of the experiments, the resultant sediment in transport does become finer, better sorted, and more negatively skewed (Table 1), as proved in the Appendix with a monotonically increasing function, suggests that t(s) does fulfill the assumption to a degree sufficient to produce the predicted trends.

The curves illustrated in Figure 3 show that, as the flow rate increases, the distribution of t(s) changes from a relatively high negative skewness to nearly symmetrical. We propose to call the negatively skewed distributions that result from relatively low flow rates low-energy functions, and the near-symmetrical distributions that result from high-flow rates high-energy functions. Because these terms are relative to the grain-size distribution of the source sediment, we can expect that the coarser a sediment is, the less likely it is to be acted upon by a high-energy sediment-transfer function. Conversely, the finer the sediment distribution, the easier it becomes for a high-energy transfer function to operate on it. In other words, the same transport process may be represented by a high-energy transfer function when acting on fine sediments, and by a low-energy transfer function when acting on coarse sediments.

The Shape of X(s)

Using these empirically derived t(s) functions (Table 2) we can now examine the possible forms of X(s), the function relating any two sequential deposits in the direction of transport (Fig. 2). We can calculate X(s) from Equation 2 using various pairs of the derived transfer functions as t1(s) and t2(s), and hence determine the relative changes in grain-size distributions between d3(s) and d1(s) by applying the theoretical results of the Appendix. For illustrative purposes we have chosen three pairs of transfer functions from Table 2 to demonstrate the form of X(s) under the following conditions, which are summarized in Figure 4.

1. t1 < t2 (energy is increasing in the direction of transport): The resultant X(s) may be calculated by letting t1 = t1 (Table 2), which is the lowest energy function, and t2 = t2, the highest energy function. As seen in Table 3 and Figure 5, the result (Xt_A1D) has a negatively skewed distribution. A similar function is derived for any of the t1, t2 pairs provided t1 < t2. Because of its negative skewness, the function is monotonically increasing over most of the sediment distribution, and, therefore, we may apply the results of the Appendix which indicate that d3(s) will be finer and more negatively skewed than d1(s). This situation may be generally unobservable in reality because d3(s) is likely to be eroded and removed by the increasing energy regime.

2. t1 > t2 (energy decreasing in the direction of transport, and t1 is a low-energy function): X(s) may be illus-
1. \( t_1 < t_2 \) (energy increasing; \( t_1 \) and \( t_2 \) either high or low energy functions)

\[
\begin{array}{c}
t_2 \\
t_1
\end{array} = \quad \text{sediment becoming finer and more negatively skewed in the direction of transport.}
\]

2. \( t_1 > t_2 \) (energy decreasing; \( t_1 \) is a low energy function)

\[
\begin{array}{c}
t_1 \\
t_2
\end{array} = \quad \text{sediment becoming finer and more negatively skewed in the direction of transport.}
\]

3. \( t_1 > t_2 \) (energy decreasing; \( t_1 \) is a high energy function; \( t_2 \) is either high or low)

\[
\begin{array}{c}
t_1 \\
t_2
\end{array} = \quad \text{sediment becoming coarser and more positively skewed in the direction of transport.}
\]

Fig. 4.—Diagrammatic summary of the resultant \( X(s) \) functions relating deposits in the direction of transport given selected combinations of \( t_1(s) \) and \( t_2(s) \).

In addition to the shapes of the transfer function \( X(s) \), the model presented in Figure 2 suggests that two other forms of transfer functions may occur. First, in the event that \( g(s) \) is known, then any deposit \( d_n(s) \) can be related to it by

\[
d_n(s) = kg_n(t_n)(1 - t_{n+1}).
\]

The above rules are not affected by this somewhat different form of the transfer function represented by \( t_n \) · \( 1 - t_{n+1} \).

Second, in the event that \( t_2 = 0 \), then \( d_2(s) \) is a final or total deposit (McLaren 1981) and can be related to \( d_1(s) \) by

\[
d_2(s) = \frac{kd_1(t_1)}{1 - t_1}.
\]

The transfer function represented by \( \frac{t_1}{1 - t_1} \) is always negatively skewed, regardless of whether \( t_1 \) is a high- or low-energy function; thus, \( d_2(s) \) will be finer and more negatively skewed than \( d_1(s) \).

APPLICATIONS

We now wish to determine if the changes predicted by the above model can be observed in natural environments for which the sediment-transport direction is known. Our purposes are to (1) establish a real-world credibility for the model, (2) provide a method to predict the pattern of sediment transport from changes in grain-size distributions, and (3) obtain information concerning the relative energy of the transport regime as summarized in Figure 4.

According to the models presented in Figures 1 and 2, grain-size distributions will change in response to erosion,
Table 3.—X(t) distributions derived from various combinations of two transfer functions (Equation 2) listed in Table 2. For example, X_{a,d} indicates t = t_a and t > t_d from Table 2. The functions are plotted in Figure 5.

<table>
<thead>
<tr>
<th>Grain Size φ</th>
<th>X_{a,d}</th>
<th>X_{a,a}</th>
<th>X_{a,c}</th>
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</tr>
<tr>
<td>-3.25</td>
<td>0.01</td>
<td>0.01</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Transport, and deposition in such a way that a lag will be coarser and more positively skewed than its source (case A), whereas sequential deposits will become either finer and more negatively skewed (case B) or coarser and more positively skewed (case C) (Fig. 6). Variance or sorting must eventually become better in each of the three cases. It is noted that case A and case C produce identical trends and that case A by itself does not provide a transport direction. The differentiation between cases A and C will depend on the geological interpretation of the environments being sampled.

**Method**

In reality, a perfect sequential change in grain-size distributions in the down-current direction, as illustrated in Figure 2, is seldom achieved due to complicating factors such as variability in the "original source" (ζ) in Fig. 2), local and temporal variability in the transfer functions, and a variety of sediment sampling difficulties (see McLaren 1981 for further discussion on sampling). Therefore, we adopt a statistical approach to determine the transport direction by examining all possible pairs in a sample suite. Given a sequence of n samples, there are \( n^2 - \frac{n}{2} \) directionally oriented pairs that may exhibit a trend suggesting transport in one direction, and an equal number of pairs in the opposite direction. When any two samples are compared with respect to their mean size, sorting, and skewness, eight possible trends exist; compared to \( d_1, d_2 \) may be (1) finer (F), better sorted (B), and more negatively skewed (−); (2) coarser (C), more poorly sorted (P) and more positively skewed (+); (3) C, B, −; (4) F, P, −; (5) C, P, −; (6) F, B, +; (7) C, B, +; or (8) F, P, +. Of these trends, only two are indicative of transport, namely F, B, − (case B), and C, B, + (case C), for which there is a one-eighth probability of either occurring at random (p = 0.125). Because of the uncertainty associated with variance, which can become larger (more poorly sorted) before becoming smaller, we choose only to accept better sorting as the criterion in the two cases suggesting the direction of transport. To determine if the number of occurrences of a particular case exceeds the random probability of 0.125, we test the following two hypotheses:

- **H_0:** p ≤ 0.125, and there is no preferred direction; and
- **H_1:** p > 0.125, and transport is occurring in a preferred direction.

Using the Z-score (Spiegel 1961) in a one-tailed test, \( H_1 \) is accepted if

\[
Z = \frac{x - Np}{\sqrt{Npq}} > 1.645 \quad (0.05 \text{ level of significance})
\]

or

\[
> 2.33 \quad (0.01 \text{ level of significance}),
\]

where \( x = \) observed number of pairs representing a particular case in one of the two opposing directions; and \( N = \) total number of possible unidirectional pairs. \( N = \frac{n^2 - n}{2} \) where n = number of samples in the sequence; \( p = 0.125; \) and \( q = 1.0 - p = 0.875.\)

The Z-statistic is considered valid for \( N \geq 30 \) (i.e., a large sample). Thus, for this application, a suite of 8 or 9 samples is the minimum required to evaluate adequately a transport direction (i.e., \( 9^2 - 9 \div 2 = 36, \) the total possible pairs in one direction).
Figure 7.—Map of the study reach in the East Fork River, Wyoming, showing the 22 bed-material sample locations (after Emmett et al. 1980).

**Example 1: Fluvial Transport**

This example uses grain-size distributions of bed-material samples from the north-flowing East Fork River, Wyoming (data calculated from Emmett et al. 1980). This river rises in the Wind River Range and flows about 50 km before arriving at the 2-km-long study reach (Fig. 7). Here the stream is about 30 m wide and meanders over a floodplain which is confined by glacial outwash terraces of sand and gravel. These terraces provide a continuous source of fresh sediment wherever the river impinges laterally against them. A series of 22 bed-material samples averaged across the channel, were collected between 43 m and 1,830 m upstream from a reference point marked by a bed-load trap (Fig. 7). The moment measures reported in Table 4 were calculated from the sand fraction only.

The 22 samples (n) provide a possible 231 (N) "north-trending" pairs and, conversely, 231 "south-trending" pairs. Of the possible cases indicative of a transport direction, only case C (coarser, better-sorted, and more positively skewed) in the north direction is significant (Table 5), demonstrating that grain-size distributions are changing in the downstream direction in a manner predicted by the model. The occurrence of the case C trend also indicates that the energy regime of the river is tending to decrease downstream, although the transfer functions must have "high-energy" shapes with respect to the sand-size distributions present in the river.

**Example 2: Delta-Lacustrine Transport**

This example uses grain-size data from Lake Tekapo, a deep, glacier-fed lake in New Zealand (data provided by R. A. Pickrell, pers. comm.; Pickrell and Irwin 1983).

The lake occupies a glacially excavated valley which is 27 km long and 6 km wide and contains a flat-floored basin below 100 m (Fig. 8). The Godley River forms a delta at the northern end, the foresets of which slope southwards to merge with the basin floor 6 km away. The lake itself is dammed in the valley by moraine and outwash deposits through which the Tekapo River flows.

An examination of grain-size trends among 25 grab samples (Table 6) taken along the axis of the lake suggests the following:

1. Of 300 possible pairs contained in the complete suite of samples, case B in the south direction is the only significant trend (Table 7). Therefore, for the lake as a whole, sediment trends predict accurately the transport direction. They also suggest that the transport processes produce low-energy transfer functions and that energy is generally decreasing in the transport direction, both of which are reasonable in this lacustrine setting.

2. For the nine delta slope samples case B is again the preferred trend (Table 7), which is also perfectly reasonable, given a decreasing energy regime with increasing depth down the delta foreslope.

**Table 4.—Grain-size data from the East Fork River, Wyoming (calculated from data in Emmett et al. 1980)**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Location (see Fig. 7)</th>
<th>Mean</th>
<th>Sorting</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>043</td>
<td>0.70</td>
<td>0.97</td>
<td>0.63</td>
</tr>
<tr>
<td>2</td>
<td>075</td>
<td>0.68</td>
<td>0.97</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>137</td>
<td>1.32</td>
<td>1.20</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>220</td>
<td>0.70</td>
<td>1.04</td>
<td>0.92</td>
</tr>
<tr>
<td>5</td>
<td>301</td>
<td>0.86</td>
<td>1.11</td>
<td>0.55</td>
</tr>
<tr>
<td>6</td>
<td>421</td>
<td>1.65</td>
<td>1.42</td>
<td>0.35</td>
</tr>
<tr>
<td>7</td>
<td>516</td>
<td>0.92</td>
<td>1.22</td>
<td>0.40</td>
</tr>
<tr>
<td>8</td>
<td>602</td>
<td>0.93</td>
<td>1.04</td>
<td>0.37</td>
</tr>
<tr>
<td>9</td>
<td>708</td>
<td>1.20</td>
<td>1.10</td>
<td>0.35</td>
</tr>
<tr>
<td>10</td>
<td>808</td>
<td>1.38</td>
<td>1.00</td>
<td>0.42</td>
</tr>
<tr>
<td>11</td>
<td>898</td>
<td>1.35</td>
<td>1.35</td>
<td>0.55</td>
</tr>
<tr>
<td>12</td>
<td>985</td>
<td>1.55</td>
<td>1.17</td>
<td>-0.07</td>
</tr>
<tr>
<td>13</td>
<td>1,077</td>
<td>0.89</td>
<td>1.04</td>
<td>0.29</td>
</tr>
<tr>
<td>14</td>
<td>1,155</td>
<td>0.92</td>
<td>1.25</td>
<td>0.66</td>
</tr>
<tr>
<td>15</td>
<td>1,241</td>
<td>0.84</td>
<td>1.15</td>
<td>0.69</td>
</tr>
<tr>
<td>16</td>
<td>1,315</td>
<td>1.74</td>
<td>1.25</td>
<td>-0.24</td>
</tr>
<tr>
<td>17</td>
<td>1,396</td>
<td>0.94</td>
<td>1.37</td>
<td>0.78</td>
</tr>
<tr>
<td>18</td>
<td>1,481</td>
<td>1.26</td>
<td>1.31</td>
<td>0.55</td>
</tr>
<tr>
<td>19</td>
<td>1,662</td>
<td>1.44</td>
<td>1.06</td>
<td>0.36</td>
</tr>
<tr>
<td>20</td>
<td>1,695</td>
<td>1.26</td>
<td>1.07</td>
<td>0.26</td>
</tr>
<tr>
<td>21</td>
<td>1,766</td>
<td>0.95</td>
<td>1.28</td>
<td>0.65</td>
</tr>
<tr>
<td>22</td>
<td>1,830</td>
<td>1.58</td>
<td>1.99</td>
<td>0.49</td>
</tr>
</tbody>
</table>

**Table 5.—Summary of the numbers of pairs of East Fork samples (Table 4) producing transport trends. N, x, and Z are defined in text**

<table>
<thead>
<tr>
<th>Case</th>
<th>North Trend</th>
<th>South Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>N = 231</td>
<td>N = 231</td>
</tr>
<tr>
<td></td>
<td>x = 29</td>
<td>x = 38</td>
</tr>
<tr>
<td></td>
<td>Z = 0.02</td>
<td>Z = 1.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>North Trend</th>
<th>South Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>N = 231</td>
<td>N = 231</td>
</tr>
<tr>
<td></td>
<td>x = 71</td>
<td>x = 33</td>
</tr>
<tr>
<td></td>
<td>Z = 8.38*</td>
<td>Z = 0.82</td>
</tr>
</tbody>
</table>

* Significant at the 0.01 level.
(3) Within the basin, case B trends are significant in both directions, suggesting the occurrence of transport in the low-energy regime. Two reasons which may be responsible for obscuring a preferred direction are sediment input from other rivers entering both sides of the lake, such as the Mistake and Cass rivers (Fig. 8), and the breakdown of the thermocline in winter, resulting in weak and random bottom currents (Pickrill and Irwin 1983).

(4) Eight samples from the south slope show an apparent case B trend in the south direction (Table 7), indicating that sediment transport is occurring upslope from below 100 m to the Tekapo River outflow. Such currents moving upslope have been observed in similar lakes (e.g., Kootenay Lake, British Columbia; C. H. Pharo, pers. comm.). It is difficult, however, to imagine that the energy regime is decreasing with shoaling water, the requirement for case B transport (i.e., $t_1 > t_2$). On the contrary, not only are higher energies associated with shoaling water, but southward currents will be concentrated into the outflow. Mathematically, it was shown that sediments also become finer, better sorted, and more negatively skewed if energy increases in the direction of transport (i.e., $t_1 < t_2$; Fig. 4), but it was argued that successive deposits would be removed by erosion with an increasing energy regime. This example tends to suggest that such deposits can remain, probably as a result of the difficulty in resuspending fine silt and clay-sized particles once they have been deposited.

There is also an apparent southward case C trend which is significant at the 0.05 level (Table 7). This is the result of sample 25 taken close to the outflow, which is coarser, better sorted, and more positively skewed than any of the other south slope samples (Table 6). Case C is unlikely as it demands a decreasing energy regime; therefore, case A is accepted as a logical interpretation whereby sample 25 is a lag of those sediments presently located in deeper water.

**Example 3: Longshore Transport**

Coburg Peninsula, a spit in the Strait of Juan de Fuca near Victoria, British Columbia, originates at its south-
west end from a low, eroding bluff composed of till (Fig. 9). The spit extends northeast about 2.5 km, where it terminates at a narrow tidal channel. The direction of spit growth and several other similar spits on both sides of the Strait of Juan de Fuca confirm a longshore transport direction which is predominantly eastwards.

A sequence of eight samples from the lower beach face and two samples from the eroding till bluff reveal that all of the beach samples are coarser, better sorted, and more positively skewed than the till (Table 8). This suggests case C transport, which is consistent with the till being the dominant sediment source for the beach. The eight beach-face samples also show a significant case C trend to the northeast (Table 9) in the direction of spit growth, illustrating not only that the model appears to have real-world validity, but can also predict correctly the direction of sediment transport.

**DISCUSSION AND CONCLUSIONS**

We have attempted to demonstrate that grain-size distributions change in the direction of transport according to the shape of the transfer function X(s). Because X(s) is the result of two transfer functions, (1), and t2(s), whose shapes can be determined empirically, we can demonstrate that sequential deposits may become either coarser, better sorted, and more positively skewed (high-energy t(s)), or finer, better sorted, and more negatively skewed (low-energy t(s)) with a decreasing energy regime. It is interesting to note that sediments cannot become coarser forever because, with coarsening, it becomes less and less likely that the transport processes will maintain high-energy characteristics with respect to the coarsening sediment. As the deposits become coarser, the transfer function describing the processes will take on the characteristics of the low-energy function, and the sediments will become finer again.

The model indicates that sediments can also become finer in the direction of transport with an increasing energy regime. This somewhat surprising result appears initially to be of theoretical value only as intuition would suggest that down-current deposits could not remain to be observed. However, one example used in the determination of transport direction suggests that such deposits can remain, possibly as a result of cohesion in fine sediments or a high-sediment-supply rate.

**TABLE 8.—Grain-size data from the Coburg Peninsula, British Columbia**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean</th>
<th>Sorting</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.36</td>
<td>1.19</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>0.27</td>
<td>0.80</td>
<td>1.29</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td>4</td>
<td>0.30</td>
<td>0.94</td>
<td>0.96</td>
</tr>
<tr>
<td>5</td>
<td>0.27</td>
<td>0.80</td>
<td>1.13</td>
</tr>
<tr>
<td>6</td>
<td>0.04</td>
<td>0.80</td>
<td>1.70</td>
</tr>
<tr>
<td>7</td>
<td>0.27</td>
<td>0.73</td>
<td>2.93</td>
</tr>
<tr>
<td>8</td>
<td>0.00</td>
<td>0.78</td>
<td>2.35</td>
</tr>
<tr>
<td>9</td>
<td>0.09</td>
<td>0.95</td>
<td>1.60</td>
</tr>
</tbody>
</table>

**Note:** Average of 2 samples.
We have also modified the earlier model proposed by McLaren (1981) which indicated that sediments always become better sorted in the direction of transport. Although not necessarily true, we suggest that in reality, successive deposits seldom become more poorly sorted, particularly when sediment distributions are described in phi units. The log-transformation tends to make grain-size distributions relatively symmetrical, a requirement that ensures a decrease in variance when the transfer function is predominantly increasing or decreasing monotonically. McLaren (1981) also suggested that sediments could become finer and more positively skewed, which this new analysis, using realistic transfer functions, demonstrates is incorrect. When sediments become finer, the skewness must become more negative (Fig. 6).

At present, we have made no attempt to explain the processes responsible for determining the shape of X(s). We have, however, demonstrated a method for the prediction of sediment transport paths, regardless of the process, that appears to give the correct directions in a wide variety of environments, even though we are using only the mean, sorting, and skewness as sediment-grain-size descriptors. It would clearly be preferable to base the prediction of a transport trend on the complete distribution of X(s), but at present there is a lack of suitable experimental data which would allow us to examine in detail all the shapes that X(s) may take. In the future we hope to utilize the complete distribution of X(s) in the determination of a transport trend and to correlate its shape with known processes and depositional environments.

ACKNOWLEDGMENTS

The writers would like to thank E. Klovan, R. E. Thomson, and R. W. Dalrymple for critically reading earlier drafts and providing the basis for considerable improvements. T. J. Day also provided advice and loaned appropriate data. The help of R. Currie for necessary computer programming and D. Chisholm for typing the manuscript is gratefully acknowledged.

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Table 9.—Summary of numbers of pairs of Coburg Peninsula samples (Table 8) producing transport trends. N, x, and Z are defined in text

<table>
<thead>
<tr>
<th>Trend</th>
<th>NE Trend</th>
<th>SW Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case B</td>
<td>F</td>
<td>N = 28</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>x = 0</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>Z = -2.00</td>
</tr>
<tr>
<td>Case C</td>
<td>C</td>
<td>N = 28</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>x = 14</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>Z = 6.00</td>
</tr>
</tbody>
</table>

1 Significant at the 0.01 level.
APPENDIX

As a first step in the analysis, we will consider the situation as outlined in Figure 1, where:

- $g(s)$ is a function of "$s$" (grain size in $s$ units) describing any grain-size distribution;
- $t(s)$ is a transfer function that is a monotonically increasing function of "$s$"; $t(s_1) < t(s_2)$ for $i < j$ and $t(s_1) < t(s_i) < 1 \lor i$
- $r(s)$ is the size distribution of sediment in transport that is derived from $g(s)$;
- $t'(s)$ is the lagging function and is a monotonically decreasing transfer function;
- $d(s)$ is the first deposit derived from $g(s)$; it may be considered a lag remaining behind after $r(s)$ has been removed.

Let

$$r(s) = k g(s) h(t(s))$$  \hspace{1cm} \text{[A]}$$

where

$$k = \frac{1}{\sum_{i=1}^{N} g(s_i) h(t(s_i))} \quad \text{(k is applied to normalize r(s))}$$

and

$$d(s) = k g(s) t'(s)$$

$$k' = \frac{1}{\sum_{i=1}^{N} g(s_i) t'(s_i)}$$

We will now apply $t'(s)$ $n$ times in order to generate $d_n(s)$, which may be considered as the $n$th lag of $g(s)$ (Fig. 10).

Then,

$$d_n(s) = k' g(s) t'(s)^n$$  \hspace{1cm} \text{[B]}$$

where

$$k_n = \frac{1}{\sum_{i=1}^{N} g(s_i) t'(s_i)^n}$$  \hspace{1cm} \text{[C]}$$

Note that for $n = 0$, $d_0(s) = g(s)$.

Consider $d_n(s)$ when $i = 1$ (i.e., the first class interval). From Equation B,

$$d_1(s) = k' g(s) t'(s)$$

Substituting for $k'$ from Equation C gives

$$d_1(s) = \frac{g(s_1) t'(s_1)^n}{g(s_1) t'(s_1)^n + g(s_2) t'(s_2)^n + \ldots}$$

Because it was assumed that $t'(s_1) > t'(s_2) > t'(s_i)$, then

$$t'(s_1) \rightarrow 0 \quad \text{as} \quad n \rightarrow \infty \lor i > 1$$

and

$$d_n(s) = \frac{g(s_1)}{g(s_1) + 0 + 0 + \ldots} \rightarrow 1 \quad \text{as} \quad n \rightarrow \infty.$$  

The converse argument, consider $r_n(s)$ when $i = N$ (i.e., the last class interval). Expanding from Equation A, as above, gives

$$r_n(s_0) = \frac{g(s_0) t'(s_0)^n}{g(s_0) t'(s_0)^n + g(s_1) t'(s_1)^n + \ldots}$$

Again, because $t(s_1) < t(s_2) < t(s_i)$, then

$$t'(s_i) \rightarrow 0 \quad \text{as} \quad n \rightarrow \infty \lor i < N,$$

and

$$r_n(s_0) = \frac{0 + 0 + \ldots + g(s_0)}{g(s_0)} \rightarrow 1 \quad \text{as} \quad n \rightarrow \infty.$$
Fig. 11.—Diagrammatic illustration of the changes in the variance of \( r(s) \) and \( d(s) \) relative to \( g(s) \). Although the variance of both \( r(s) \) and \( d(s) \) must ultimately approach zero, the location of \( n = 0 \) and the variance of \( g(s) \) may result in \( \sigma_r^2 \) or \( \sigma_d^2 \) increasing before their eventual decrease. In this illustration, \( \sigma_r^2 \) has increased at \( n = 1 \).

then

\[ r_n(s_0) \to 0 \quad \text{as} \quad n \to \infty \quad \text{for} \quad i < N \]

and

\[ r_n(s_N) \to 1 \quad \text{(Fig. 10)}. \]

**Mean Grain Size**

\[ \mu_r = \sum_{i=1}^{N} r_i(s_i)s_i, \]

where \( \mu_r \) is the mean grain size of \( r(s) \), the distribution of sediment in transport (Fig. 1).

Let

\[ \mu_r = \frac{1}{N} \sum_{i=1}^{N} g_i s_i \]

which, by applying Equation A, becomes

\[ \mu_r = \frac{1}{N} \sum_{i=1}^{N} g_i (s_i - \mu_g)^2 s_i. \]

As

\[ n \to \infty, \]

\[ \mu_r \to s_n \quad \text{(Fig. 10)}. \]

Because

\[ \mu_r < s_n, \]

then

\[ \mu_r > \mu_g \quad \forall \ n. \]

This proves that, if the transport function \( t(s) \) is increasing monotonically, then sediment in transport must be finer than its source. By the converse argument:

\[ \mu_r < \mu_s \quad \forall \ n \quad \text{(Fig. 10)}. \]

Thus, the first deposit (or lag) must be coarser than its source.

**Variance (Sorting)**

\[ \sigma_r^2 = \frac{1}{N} \sum_{i=1}^{N} (s_i - \mu_g)^2 g_i \]

where \( \sigma_r^2 \) is the variance of \( r(s) \), the grain-size distribution in transport (Fig. 1). Let

\[ \sigma_r^2 = k_n \sum_{i=1}^{N} (s_i - \mu_r)^2 g_i(s_i - \mu_g)^2. \]

**Skewness**

\[ \text{Sk}_r = \frac{1}{(\sigma_r)^{3/2}} \sum_{i=1}^{N} (s_i - \mu_g)^3 g_i, \]

where \( \text{Sk}_r \) is the skewness of the source sediment. Similarly,

\[ \text{Sk}_r = \frac{1}{(\sigma_r)^{3/2}} \sum_{i=1}^{N} (s_i - \mu_r)^3 r_i(s_i), \]

where \( \text{Sk}_r \) is the skewness of the \( r(s) \) distribution.

As \( n \to \infty \), let

\[ r_n(s_0) \to 1 - a, \]

\[ r_n(s_{N+1}) \to a, \]

and

\[ r_n(s_i) \to 0 \quad \text{for} \quad i = 1, 2, 3, \ldots, N - 2, \]
where "a" is arbitrarily small (Fig. 12).
Then
\[ \mu_n \rightarrow s_n(1 - a) + s_{n-1}a \]
\[ - s_n + a(s_{n-1} - s_n) \]
and
\[ \sigma_n^2 \rightarrow (s_n - \mu_n)^2(1 - a) + (s_{n-1} - \mu_n)^2a \]
\[ - a(s_{n-1} - s_n)^2. \]
Thus,
\[ (s_i - \mu_i)^2r_x(s_i) \rightarrow a(s_{n-1} - s_n)^2. \]
and
\[ \text{Sk}_n \rightarrow \frac{a(s_{n-1} - s_n)^3}{[a(s_{n-1} - s_n)^2]^{1/2}} \]
\[ - \frac{1}{a} \]
\[ - \infty \text{ as } a \rightarrow 0. \]
\[ \therefore \text{Sk}_n < \text{Sk}_q \forall n. \]

Similarly, as
\[ n \rightarrow \infty, \]
\[ \text{Sk}_d \rightarrow +\infty; \]
\[ \therefore \text{Sk}_d > \text{Sk}_q \forall n. \]

We have shown, therefore, that the skewness of sediment in transport must become more negative than the source sediment, and the skewness of a lag must become more positive.

**Summary**

Given a monotonically increasing transfer function (in \( \phi \) units), it has been shown that sediment in transport must become progressively finer and more negatively skewed than its source sediment. Conversely, the lag must become coarser and more positively skewed than its source sediment. The change in sorting (variance) cannot be determined in the general case, although at some point in the transport path sorting must become better.